CLIMATE MATHEMATICS

MCRN

Mathematics and Climate Research Network

Mathematics in Climate Science: Models and Data

Why do we model the climate system?

Carrying out experiments on the entire climate system is unfeasible. This is because of the simple fact that we have only one Earth and it is in use! Creating a mathematical replica of the Earth in the form of a climate model is the only way to carry out unlimited experiments. The big climate models, at the thirty or so main climate science centres around the world, can thus be seen as the *laboratories* of climate scientists.

What are the models used for?

They allow scientists to ask questions like: Can we expect more droughts in California in a warming climate over the next two decades? Will the Arctic be free of summer sea ice in the near future? Will weather patterns in Northern Europe change significantly? The interest is now in decadal prediction[1], which is to say predicting what will happen in a warming climate over the next few decades. It is viewed as an extension of weather prediction but with a crucial difference; instead of forecasting specific events in days or weeks, we want to predict averages over years and decades.

Are the models used to predict the extent of warming?

The Intergovernmental Panel on Climate Change (http://www.ipcc.ch/) is the main world-wide group formulating the physical science basis for our understanding of climate change. Up until their fourth report in 2007, the focus was on very long term prediction and this corroborated both the predictions of a warming world and its anthropogenic causes. It was then decided that the case had been made and that the focus should shift to support decision-makers who will be concerned with adaptation to a changing climate and mitigation of its worst impacts.

But are the models not key in making the case that the climate is warming?

Yes, but perhaps in an unexpected way. The case that the Earth is warming due to anthropogenic emissions is based on four facts:

(1) The signature of an increase in global average temperature in observations is now unequivocal;

(2) The amount of CO_2 in the atmosphere has been measured and is increasing at a steady rate (it is now over 400 parts per million – a figure unprecedented in human history);

(3) CO_2 from natural sources can be distinguished by carbon isotope from CO_2 resulting from the burning of fossil fuels; both can be measured and the increase is due to the anthropogenic emissions;

(4) The effect of CO_2 in the atmosphere as a greenhouse gas is well understood and it is known how the gas absorbs and emits long-wave radiation, which has the effect of trapping heat.

There is no model prediction needed, but there is one extra piece of the puzzle that makes the argument watertight. If the models are appropriately initialised and run over the 20th century in two scenarios, one with the CO_2 emissions that actually happened and the other with only natural emissions of CO_2, the former captures the warming that actually occurred and the latter does not. It was actually this model experiment that convinced Jim Hansen and led him to testify to Congress in 1988 about global warming. But note it is not a prediction. The models do all predict warming over the 21st century, which is consistent with the case for global warming, but their predictions are not needed for the case to be made.

Are the models also used for understanding how the climate works?

The answer to this key question is definitely affirmative, but there is a qualification. The models are used to test scientific hypotheses and answer questions as to how the climate will react under certain scenarios, particularly involving different levels of carbon emissions. We get significant insight into the workings of the climate from this analysis as we learn what happens under different circumstances. But the whole system is so complex that it is it very hard to decipher explicit connections between different phenomena. In particular, making conclusions as to *cause and effect* is almost impossible in such a model.

To achieve this level of insight requires an isolation of the key physical effects at play. Simpler models that strip away extraneous effects and focus on the mechanisms of interest are very useful here. For instance, biogeochemistry is concerned with how different chemical and biological species interact in an ocean environment. This is critical for the overall climate as it dictates how carbon is stored in the ocean. It is studied by isolating the key chemical reactions and placing them in a physical (fluid) context that captures some of the key features of the ocean environment.

So, there are models of all shapes and sizes: How do they fit together?

In current practice, there are two ways in which the models that focus on a restricted part of the physics play a role in the formation and functioning of the full-scale climate models. The first is through the process of parametrisation. Since the full models must necessarily omit certain processes, by virtue of their being at too small a scale or being too detailed and complex, they need to account for their effect on the variables captured by the full model. This is done by running models that capture the missing process offline, and estimating the parameters that reflect them and appear in the full model.

The second is through the building of expert opinion. The big models are judged on whether they capture behaviour that is expected based on the understanding that climate scientists have about the way certain key parts of the climate behave. This expert opinion is built up over time and much of it depends on observations of the climate system, but a good deal is also built on how paradigmatic models behave. The full-scale models are then tuned and this involves another resetting of parameters to ensure model fidelity to this pre-existing knowledge.

Are there major climate phenomena that are difficult, or even impossible, to study using the full models?

The big models have had their rough edges smoothed out by ensuring that all the complex numerical computations work well together and the models are well calibrated to capture the observed climate from the 20th century. They are finely tuned machines and are, as a result, unlikely to produce any drastic changes. Like luxury automobiles, they are designed to be conservative in their reaction to any stimulus. But we know that the climate may, and has in the past, undergone abrupt shifts. Many dangers lurk in our current climate and, although some are considered fanciful by experts such as the shutting-down of the Gulf Stream, others are real possibilities. These include eventualities involving sea and land ice, such as destabilisation of the Antarctic Ice Sheet and melting of permafrost in the tundra that could cause the release of large quantities of methane. But ice is not well understood, nor are its characteristics well captured by the big models.

What are some examples where a mathematical look at simpler models can give important insight?

I will give two, one by a British group[2] at Exeter – although a key member is now at Cork in Ireland – which is part of the UK network on mathematics in climate science (CliMathNet) and another by an American group[3] (of which I am a member) that formed through the

Mathematics and Climate Research Network (MCRN). The Exeter group exposed a new phenomenon in the study of abrupt change. Their initial work showed that a *compost bomb* (sudden ignition of a peat bog) could happen under conditions that looked stable. The cause would not be the temperature increase itself but the rate at which it was increasing. Their insights now go beyond this particular situation to show the relevance of this phenomenon in warming climate as it may not be the popularly-discussed temperature increase over the pre-industrial era – which was the focus of the Paris Climate Accord – but the rate at which we get there that is critical. The American group has studied El Niño through a stripped down model. The question is what causes the false alarms with mini El Niño's which fade out. Last year (2014-15) there was such an event. The group has shown there is a mathematical effect that occurs when the system is governed by two (or more) different time scales – in this case those of the ocean and atmosphere. This effect produces exactly this type of near-miss in triggering El Niño.

Shouldn't we be able to see these in a full climate model?

It may be possible, but the odds are against it for various reasons. Complexity is a two-sided coin. On the one hand, the system is most likely complex enough that there are some conditions under which something seen in a simpler model may happen. But, on the other hand, the search for those conditions can be overwhelming. This points to one of the greatest challenges we face, and it is essentially a mathematical one. The full models and the stripped-down models are at two ends of a hierarchy of models with increasing complexity. How dynamical phenomena seen at one end can be carried up the hierarchy is not at all well understood.

What about data? Aren't there enough observations now to make this modelling obsolete?

Observational data are obviously key to mapping out what is happening in the climate, what will happen and understanding why it all happens in the way it does. But the data are not enough on their own, and never will be. There is the issue of how to organise all the data we have; this is the problem going under the umbrella *Big Data* and occurs in all areas where data have proliferated. The more data we have, the harder it becomes to organise into something coherent and informative.

But this isn't the main issue with climate. However many data we collect, they will never be enough to give a full picture of the climate. Imagine just the simple problem of mapping the temperature distribution in a fixed region, say across the UK, at any given time. We can do a pretty good job by measuring at well-placed stations and then interpolating between them. But we know there can be microclimates and variations that might get missed. This may not matter as the variations most likely won't be that great. But what if, for some reason, we needed an extraordinary level of accuracy? Suppose we also want to know wind speed at each location, and barometric pressure, which both might have greater variation. We could argue that you just need to get more and more measurements to get more accuracy. This may be true, but it may not because we also know that small variations might lead to bigger ones later on. The problem lies in the idea that interpolation between data points can be done crudely. Indeed, there is knowledge we have that is not being used here. The values of these variables (temperature, wind speed, pressure etc.) must be consistent with the laws of physics. And models are what encode the laws of physics and should be used to constrain the assignment of these variables in between measuring points.

But aren't we mainly concerned with prediction?

The above consideration of how to use data was an example of how a model is needed to fill in the gaps. The need for a model becomes even more acute when we consider prediction. To get a complete picture of a given variable distribution at a fixed time involves *interpolation*, to predict into the future purely from data is *extrapolation*. Done using purely observational data would involve making the assumption that the future will be like the past. This may be true, but a much better approach is to use the laws of physics which tell us how the future will look given the present state. And this is exactly what a mathematical model does.

It seems that models and data offer different approaches, can we get the most out of both?

In my mind, this is the hardest and most important challenge we have facing us in studying the climate, and it is essentially a mathematical issue. We need to see models and data as on equal footing and optimise the information from each by finding the appropriate balance. They each bring to the table something critical; data bring information for the world as it actually occurs and the models bring our well informed knowledge as to how it works. The process by which data and models are balanced is called *Data Assimilation*, a subject that has primarily been developed for weather prediction. It is used in parts of climate science, but not systematically and this will be a growth area in the near future.

What are the greatest challenges involved in making Data Assimilation effective for climate studies?

The climate models are very large in dimension, which is to say that there are many variables and many physical locations where they are being determined. The systems involved in the climate are also very nonlinear, which is to say that they do not necessarily respond commensurately with an increase in stimulus. We do not know how to assimilate data into models that are both nonlinear and high-dimensional. We have effective techniques for low-dimensional, nonlinear systems and high-dimensional, linear systems, but not both! I anticipate that this will be one of the major foci of mathematical research in climate over the next decade. It is a problem that we badly need to solve.

1. Decadal Climate Prediction: an update from the trenches – Gerald A. Meehl et al. Bulletin of the American Meteorological Society (2014) 95, 243-267 doi:10.1175/BAMS-D-12-00241.1

2. Tipping points in open systems: bifurcation, noise-induced and rate-dependent examples in the climate system – Peter Ashwin, Sebastian Wieczorek, Renato Vitolo and Peter Cox Phil. Trans. R. Soc. A (2012) 370, 1166–1184 doi:10.1098/rsta.2011.0306

3. Mixed mode oscillations of the El Niño-Southern Oscillation – Andrew Roberts, Esther Widiasih, Axel Timmermann, Christopher K.R.T. Jones and John Guckenheimer. To appear in the Journal of the Atmospheric Sciences (2016), see: http://arxiv.org/abs/1511.07472

About the Author

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Mathematics and Climate Research Network

Enabling an Emerging Research Area Using the Web

The goal of the Mathematics and Climate Research Network (MCRN) is to develop the area of "climate mathematics". A critical mass of researchers has been brought together by MCRN to forge strong bonds between groups at different locations. An infrastructure has been built that has leveraged the advanced communication capabilities of the web, including broadcast courses and webinars, web-conferencing, and small group collaborations facilitated by shared whiteboards. MCRN serves as a model for how a new area can be defined and energised in a relatively short period of time. In particular, MCRN provides:

- Access to a broad range of expertise not available on a student's home campus;
- Support and mentoring by extended group;
- Feeling of community of people with shared goals;
- Ability to forge new collaborations quickly and choose from large group of potential collaborators;
- Access to resources needed to get into a new area; and
- Broadly-based support for seeking new opportunities.

The distributed nature of MCRN has brought together a range of experts to be part of training students at the boundaries of the different subjects needed for climate mathematics. Trainees at all levels, from undergraduate to post-doctoral, are deeply involved in the Research Focus Groups, both in their organisation and spin-off research efforts. Some of the key advantages of the MCRN approach, often cited by students, are:

- Support of senior personnel from other campuses;
- Available resources through MCRN;
- Engagement throughout the year (not just a summer programme) with faculty and peers on projects; and
- Community that offers contact with many people who share similar interests.

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www.mcrn.hubzero.org

CliMathNet (Climate Mathematics Network) was founded in 2012 with support from the UK EPSRC (EP/K003216/1) and partner organisations. The network, aligned to the international Mathematics for Planet Earth 2013 initiative, aims to bring together climate scientists, mathematicians and statisticians to answer key questions around climate modelling and to identify areas of mathematics and statistics that need to be developed for this purpose.

Climate science ranges from numerical weather prediction to geochemical and biological modelling, and a wide range of mathematical research already underpins the science discussed in the Intergovernmental Panel for Climate Change (IPCC) assessment reports. It aims to promote research by recognising (a) the fundamental importance of mathematics in climate science and (b) the existence of barriers (such as different departments, different funding councils) to the science being developed in an optimal way.

CliMathNet support researchers in mathematics, statistics and climate science to get informed through a regular newsletter, to match their expertise via its directory of members and to disseminate their work through an annual scientific meeting. The network also work with policymakers to identify specific issues that can benefit from input from CliMathNet members. Finally, CliMathNet inform schools and the general public about the importance of mathematics and statistics in this area through a range of activities, including lesson plans about their fundamental role in modelling and predicting weather and climate.

The network welcomes members from academic staff, researchers, PhD students, policymakers, and those who are interested in the network. See the website for more information or to subscribe to the newsletter.

http://www.climathnet.org/

Research on Changes of Variability and Environmental Risk. This project aims to research, develop and apply new mathematical tools and methods to better understand the connections between environmental variability and environmental risk. In particular, many of the more costly impacts are a result of variability of environmental variables (such as regional rainfall) rather than averages.

We are funded by the UK EPSRC (EP/M008495/1) from 2015 to 2018 to promote research in this area via a number of mechanisms, including the funding of feasibility studies. We are one of four networks supported by EPSRC to promote groundbreaking cross-disciplinary collaborations that are developing new mathematics to address issues related to environmental change and environmental extremes. ReCoVER is closely affiliated to the research network CliMathNet and is led by Professor Peter Ashwin and Professor Tim Lenton, supported by an expert panel and an advisory board. The three focus areas of our scientific work are: (i) computing complexity, (ii) extreme events, tipping points and quantifying uncertainty, and (iii) modelling coupled social-environmental systems.

During its first year the ReCoVER Network has:

- Organised four interdisciplinary workshops, supported a further two workshops as organised by partner networks and has been represented at several more workshops and events;
- Funded 14 feasibility studies of various sizes across the scientific focus areas;
- Promoted Outreach Activities through a Virtual Outreach Conference; and
- Supported CliMathNet through their annual scientific research meetings.

Contact the project via www.recoverlwec.org if you want to know more!

Training the Future Leaders of MPE

Mathematics of Planet Earth (MPE) is a burgeoning topic which has gained considerable momentum throughout the world during the past few years. Large organisations such as the American Physical Society (APS) and the Society for Industrial and Applied Mathematics (SIAM) have established activity groups in MPE and are in the process of holding annual full scale meetings on this topic. Funding at national level has also been devoted to establishing networks of scientific communication in this topic.

Quantifying uncertainty in long-term climate prediction and estimating the potential strength of extreme meteorological events in view of global change are very difficult research areas, requiring collaborations among qualified researchers, access to massive high quality data sets, and a long-term commitment. The number of researchers in these areas is limited, and demand is growing fast. World-wide interest in MPE has increased massively, because it is being universally recognised as being both timely and necessary. Indeed, the world mathematical community has no choice but to get involved and play a substantial role in research matters focused on climate change, weather variability, ocean circulation and sustainability. However, while there is an urgent need for mathematical tools in MPE, it is not easy for a mathematician to learn and get involved in these issues.

Our response at Imperial College London and the University of Reading to this world-wide demand has been to establish a new EPSRC Centre for Doctoral Training (CDT) in MPE for the purpose of training the future leaders of mathematical research in MPE. The Centre offers students the chance to take a purpose-developed Masters by Research (MRes) in their first year, leading to a PhD project in the following three. The aim of the MRes is to give students time to develop all of the technical skills and background knowledge required to carry out their PhD work successfully. Students also receive a bespoke set of transferable skills training, designed to facilitate cohort formation, and networking opportunities, and to underpin multidisciplinary research. Collaboration with external partners is also encouraged, bringing students close to world-leading weather and climate services (such as the Met Office) and representatives from key commercial sectors, including energy, water, marine and insurance.

Moreover, the CDT is dedicated to cohort learning in which, each year, small groups of about fifteen mathematics PhD students with different backgrounds are recruited and taught to communicate with each other as they learn the diverse fundamentals of the MPE topic as a group. Each year's cohort works together as peers and friends who share the effort of learning such diverse material and develop the capability to perform research in this area, staying in contact with each other as they perform the work required to earn a mathematics PhD in this topic. The MPECDT is the first response of this type to the world-wide demand, and you can read more about our activity at the MPECDT.org website.

"Mathematics is not the traditional recruiting ground for the environmental sciences but it's an area that produces people with exactly the skills we need to understand the climate and predict the future state of the ocean and the atmosphere."

Dr David Ham, NERC Independent Research Fellow, Department of Mathematics

"The purpose of the MPE CDT is to produce PhD researchers capable of undertaking independent world-class multidisciplinary research. Typically, they are high achievers with a can-do attitude and a real enthusiasm for the topic."

Dr Anna Radomska Botelho Moniz, Centre Manager, EPSRC Centre for Doctoral Training in the Mathematics of Planet Earth

Following on from COP21

At the public session of the Environment Council in March, EU Commissioner for Climate Action and Energy, Miguel Arias Cañete delivers a follow up speech to COP21...

It is a pleasure to address you today following the adoption of the Commission's Communication on the Paris Agreement and its implications for EU climate and energy policy.

I would like to thank Minister Dijksma and the Presidency for preparing their headline reading of the Paris Agreement which is also important for today's discussion.

We have already reflected on the historic achievement of Paris and the opportunity the global low carbon transition presents. But much work lies ahead. In our Communication adopted, we set out our initial reflections on implementing the Paris Agreement in the EU. Our message is clear: the EU needs to continue to show global leadership.

Long-term goal

The Paris Agreement includes a long-term goal to put the world on track to limit global warming to well below 2°C above pre-industrial levels, and to pursue efforts to limit the temperature increase to 1.5°C.

It also contains a dynamic mechanism to review ambition over time. The 2018 facilitative dialogue presents a first opportunity to assess collective efforts in achieving the long-term goal before the first global stocktake in 2023.

The special report by the Intergovernmental Panel on Climate Change on the specific policy implications of a 1.5°C goal will be critical in informing those discussions. The EU will provide input into the work that will be undertaken internationally for that purpose.

Along with other Parties, the EU is also invited to communicate by 2020, its mid-century long-term low emission development strategies. To facilitate the preparation of these strategies, and the political debate in the Parliament and Council, the Commission will prepare an in-depth analysis of the economic and social transformations required for this purpose.

In this context, some argue that the EU needs to raise its level of ambition already today. Let me give you my view:

The EU 2030 target is an ambitious contribution to the Paris Agreement, as part of a global effort, and in comparison to other Parties contributions. This is because:

- It is consistent with what science requires. Reducing our emissions by at least 40% by 2030 is in line, in the medium term, with the Paris Agreement's goals;

- It is the most ambitious form of mitigation target. It is an economy wide cap, binding in European law, and accountable down to each tonne;

- It will require a significant transformation of the European economy. In the energy sector, transport, buildings, agriculture. In fact this concerns all sectors. It will not be easy, but we are determined to deliver;

- And importantly, it is designed as a first step in a roadmap towards a low carbon economy. As I just mentioned, the EU will prepare, by 2020, a mid-century low emissions development strategy, that will enable the EU to lead the global transformation towards climate neutrality.

2030 climate and energy framework

Coming back to the Paris Agreement, a key element is the legally binding obligation on all Parties to pursue domestic mitigation efforts necessary to achieve their emission reduction targets.

The EU was the first major economy to submit its intended nationally determined contribution (INDC) on 6 March 2015 which reflects the European Council Conclusions of October 2014 on the 2030 climate and energy framework.

We must live up to our commitments made in Paris and implement swiftly the EU's climate and energy framework, as agreed by the EU leaders in October 2014.

The Commission began the process of implementation even before the Paris Conference with its proposal to revise the EU emissions trading system (ETS).

As a next step the Commission is currently preparing proposals for the non-ETS sectors, including on effort sharing and land use, land use change and forestry, as well as a new governance mechanism to streamline planning and reporting requirements for the post-2020 period. This will also cover energy policy, with the revision of the energy efficiency and renewable energy directives as well as our work on the electricity market design scheduled for autumn.

Low carbon transition

The Paris Agreement has sent a clear signal to stakeholders and investors that the global transition to a low-carbon economy and clean energy is here to stay.

We should reflect for a moment that being in the lead on the drive towards a low carbon economy is a clear opportunity for our economy, for jobs and growth here in Europe.

To support this transition, we need an enabling framework to a real long term transition to a low carbon economy, in particular by delivering on the Energy Union.

With the scope of needed investments, it is clear that shifting and scaling up private investment is essential. EU funds, such the European Fund for Strategic Investments, will play an important role to mobilise the markets.

Indeed the EU as a whole is well placed to exploit these new opportunities. I believe that with our strong track record and continued focus on innovation, Europe is on the right track.

Climate Diplomacy

The Paris Agreement was without a doubt a great success for EU climate diplomacy. The EU spoke with a unified voice which was crucial in the lead-up to Paris and in developing the High Ambition Coalition – the alliance of countries that fought for a high level of ambition – which shaped the successful outcome.

We must maintain this momentum in all international fora including the G7 and G20, not least because of the critical negotiations in the International Civil Aviation Organization (ICAO) later this year as well as further talks on the Montreal Protocol, both of which provide a good opportunity to scale up the level of ambition in the pre-2020 period. We must also ensure tangible progress on many of the left-overs from Paris before the Marrakech conference.

As a first step I welcome the Conclusions adopted by the Foreign Affairs Council on 15 February, which recognise that climate action is amajor strategic foreign policy challenge in a range of areas with implications in development aid and cooperation, trade and security.

We should continue to support developing countries in the implementation of their national climate action plans, with such support programmes to be rolled out as of this year. In that regard, we (the EU and the Member States) should deliver on our commitment to scale up the mobilisation of climate finance in order to contribute our share of the €100bn per year by 2020. ■

Taken from a speech on the EU Commissioners website:
http://europa.eu/rapid/press-release_SPEECH-16-586_en.htm

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European Commission
http://ec.europa.eu/commission/2014-2019/arias-canete_en

MCRN: A Network Linking Researchers across the Globe to Develop the Mathematics Needed to Better Understand Earth's Climate System

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www.mathclimate.org